

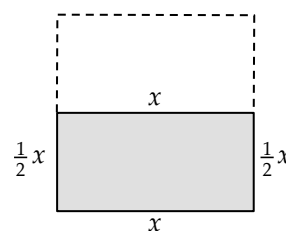
Primary Mathematics Challenge – November 2015

Answers and Notes

These notes provide a brief look at how the problems can be solved. There are sometimes many ways of approaching problems, and not all can be given here. Suggestions for further work based on some of these problems are also provided.

P1 C ($3 + 4 = 7$) P2 E ($£20 \div 50p = 40$)

- 1 C 3 m Options A, B and D represent very small elephants, and option E is huge!
- 2 A 0 We can observe that ONE, TWO and SIX have 3 letters; FIVE and NINE 4 letters; THREE, SEVEN and EIGHT 5 letters.
- 3 A 3 am Eight hours behind 11 am is 3 am.
- 4 E 12 Twelve octopuses need 96 socks, while 13 octopuses need 104 socks.
- 5 C 10p Each pencil costs 1p less than £1, so for 10 pencils Rabbit will get 10p change.
- 6 D 1338 Samuel will run the 669 steps twice; $2 \times 669 = 1338$.
- 7 C 3×4 Option A is not divisible by 4; option B is not divisible by 3. Options C, D and E are all divisible by 2, 3 and 4; C is the smallest.
- 8 E 12 Cuboid X has 3 unit cubes, whereas cuboid Y has $6 \times 2 \times 3 = 36$ cubes.
- 9 E 1.9 m Three times the circumference of Pythagoras' head is $0.63 \text{ m} \times 3 = 1.89 \text{ m} \approx 1.9 \text{ m}$.
- 10 D 50 *Floppy Bird* has a 90° sector, ie. one quarter of the chart; so we have $200 \div 4 = 50$.
- 11 B Tuesday Three weeks is 21 days, so 20 days is a day short of 3 weeks later, which is Tuesday.
- 12 C 26 There are 50 flats from 250 to 299 inclusive, and for these here Pat and Qwong deliver 25 leaflets each; Pat will also deliver to 300, so 26 leaflets.
- 13 D 64 cm^2 Let the side-length of the square be $x \text{ cm}$. The diagram on the right shows the perimeter of the rectangle is $3x \text{ cm}$. Hence $x = 8$, and so the area of the square is $8 \times 8 = 64 \text{ cm}^2$.
- 14 D 36 The first time reflected is 1702, the second 1738. Hence 36 minutes have passed.
- 15 B 5 The next years are MMXVI, MMXVII, MMXVIII, MMXIX. However, in 2020 the year will again have just 4 letters: MMXX.
- 16 A 11 The sequence of numbers Robin crosses out is as follows:



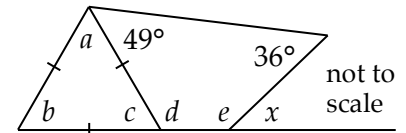
~~1~~ 2 3 ~~4~~ 5 6 ~~7~~ 8 9 ~~10~~ 11 12
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~~1~~ ~~2~~ (3) ~~4~~ ~~5~~ ~~6~~ ~~7~~ (8) ~~9~~ ~~10~~ ~~11~~ ~~12~~

leaving only 3 and 8 uncrossed; $3 + 8 = 11$.

- 17 C 42 The ratio 2 : 5 means that 2 in 7 beads are blue and 5 in 7 purple. Since $98 \div 7 = 14$, there are 28 blue and 70 purple beads, hence $70 - 28 = 42$ more purple than blue.
- 18 B 11.33 The tasks take a total of $40 + 7 + 35 + 11 = 93$ minutes = 1 hour and 33 minutes.
- 19 B 16 cm^2 The left-hand triangle has an area of $4 \times 5 \div 2 = 10 \text{ cm}^2$, and that of the right-hand triangle is $3 \times 4 \div 2 = 6 \text{ cm}^2$. So the area of the quadrilateral is $10 + 6 = 16 \text{ cm}^2$.
- 20 D 40 The number of moaning mornings is $\frac{2}{3} \times 12 \times 5 = 40$.
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- 21 0.5 kg or $\frac{1}{2} \text{ kg}$ The two cats will together weigh as much as 12 kittens. So 10 kg is equivalent to the weight of $12 + 8 = 20$ kittens. Hence one kitten weighs $10 \text{ kg} \div 20 = 0.5 \text{ kg}$.

22 150 g With each of the three dimensions halved, the volume will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ of its former volume; so the smaller box will contain $1200 \div 8 = 150$ g.

23 25° Because the triangle is equilateral, each of the angles a , b and c are 60° . Angles c and d are together on a straight line and so sum to 180° ; so $d = 120^\circ$. Angles inside the quadrilateral add up to 360° , and so $e = (360 - 49 - 36 - 120)^\circ = 155^\circ$. So angle $x = (180 - 155)^\circ = 25^\circ$.



24 $\frac{1}{3}$ We can see that the three-digit numbers ending in 6 begin at 106 and end at 996, forming a set of 90 numbers. If we explore the first ten of these (106, 116, 126, ..., 186 and 196), we can see that every third number is divisible by 6 (as the sum of the digits is divisible by 3 and the numbers are even). Therefore, since 90 is a multiple of 3, there are 30 numbers ending in 6 which are multiples of 6, so the fraction is $\frac{30}{90} = \frac{1}{3}$.

25 6277 We can check 87878 for prime factors. As 87878 is even it is divisible by 2, giving us 2×43939 . We can see from the sum of its digits that it is not a multiple of 3, and by inspection it is not a multiple of 2 or 5. However, 43939 is divisible by 7, giving us 6277. But 6277 is not divisible by 7 and the next prime number is 11, which, if it was a factor, would reduce it to a three digit number. Since we are told that 87878 has a four digit factor which is prime, it must be 6277.

Some notes and possibilities for further problems

- 2 A loosely related question is to find the unique number which spelt out as an English word has its letters in alphabetical order.* In French and German there are several, though in Spanish only one.
- 3 Allocating time zones across the world is not as straightforward as students might think. A map at <http://www.worldtimezone.com> shows this, particularly around the Pacific island of Kiribati.
- 6 In March 2015, the Australian athlete Suzy Walsham managed to run up all 1700 steps of the Eiffel Tower in 9 minutes 44 seconds. *Vertical running*, as the sport is called, is a long-established one; the *New York Times* reported a race to just the second stage of the Tower in November 1919 – then the first of 120 athletes took 3 minutes and 4 seconds.
- 13 Another square piece of paper is folded in *four* with all folds parallel. If the perimeter now is 25 cm, what was the area of the original square?
- 14 It seems quite difficult to get English words from horizontal reflections of a digital display, let alone ones which represent 24-hour clock times – perhaps, best is the voice-recognition program 1712 to read as 5117. Thinking outside of clock times, one can get 80551E7 from the number 7312208. However, rotating a digital display a half-turn, as schoolchildren with calculators have observed for well over a quarter of a century, gives much more scope for words to be formed – in fact, the mind 5376608 !
- 15 After 2020, what will be the next four-letter Roman numeral? How many more might a student now in Year 6 see in their lifetime? Perhaps such a student might try adding all of them together!
- 16 This problem is related to what has become known as the *Josephus Problem*. Josephus Flavius was a famous Jewish historian in the first century AD. According to legend, he and a band of soldiers became trapped in a cave surrounded by Romans. Preferring suicide to ignoble capture, the Jews formed a circle and proceeded around it, killing every third remaining person until only two were left. Josephus, by chance (or otherwise), was able to get into the safe last two spots and so stayed alive. A mathematical simulation of this (without any gruesomeness) can be found on the excellent site <http://www.cut-the-knot.org/recurrence/flavius.shtml>.
- 23 The sizes of the angles in degrees 25, 36 and 49 are square numbers – is there another problem where many (or all) of the angles (given or calculated) can be measured in square numbers of degrees?
- 25 Does the answer change if one considers four-digit numbers, or even larger? What about a similar question for other digits – for the digits 1, 2 or 5, the answer is obvious, but what about multiples of 7 ending in 7?